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Signals from the Future†

A. PERES and L. S. SCHULMAN‡

Department of Physics, Technion—Israel Institute of Technology, Haifa

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Abstract

Advanced interactions do not lead to dynamical inconsistencies in closed systems. They are, however, incompatible with the existence of truly random processes.

1. *Formulation of the Problem*

Heat usually flows from hot to cold bodies. However, there are sophisticated machines (e.g., refrigerators) where heat is made to flow from cooler to warmer regions. Likewise, moving charges usually interact by their *retarded* fields. A signal sent by an emitter is observed by a receiver only at a *later* time. However, one may ask whether it is possible to construct more sophisticated emitters where effects would be given by *advanced* fields, so that signals would be received *before* they are emitted. More generally, one may ask whether there is anything *in principle* forbidding information to be carried from future to past. Certainly there can be no such principle in a theory whose equations and boundary conditions are time reversal invariant. The time reversal operator can then produce a physically valid universe with both heat and information flow reversed. The question we pose here, however, is for *our* universe, where such a symmetry does not appear to exist in the large.

There has recently been a renewal of interest in this question because, if tachyons (Bilaniuk *et al.*, 1962; Feinberg, 1967) exist, and moreover if they can be emitted and received in a *controllable* way (Peres, 1969), then they can be used as the agent conveying information from future to past.§ The purpose of this note is to discuss which fundamental principle, if any,

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‡ NATO Fellow. On leave from: Physics Department, Indiana University, Bloomington, Indiana 47401.

§ Many models have been constructed whereby an observer can send information to his own past with the help of tachyons: see Peres (1969), Rolnick (1969), Pirani (1970), Benford *et al.* (1970), Newton (1970).

would be violated if information is conveyed (by tachyons, or otherwise) from future to past.

The usual argument is that if one gets information from the future he can act in such a way as to contradict that information. For instance, if one has an emitter programmed to send a signal at 3 p.m., and the signal is received at 1 p.m., he can decide to destroy the emitter at 2 p.m., if and only if the signal has been observed at 1 p.m. This system is intrinsically inconsistent.

2. *The Solution Proposed by Wheeler and Feynman*

This paradox was analyzed long ago by Wheeler & Feynman (1949) who proposed a mechanical model with the properties described above. It turned out that the model did have a consistent motion: roughly speaking, the signal was very weak, so that the mechanism destroying the emitter acted after a long delay, just in time to let the emitter send a very weak signal.

More recently these continuity based arguments were applied to resolve paradoxes associated with tachyons (Schulman, 1971).

A simple model, showing the essential features of the problem, consists of a particle with momentum p receiving at time $t = 0$ a large impulse of magnitude K, the sign of which is *opposite* to the sign of p at some *future* time τ. Suppose the initial value of p (i.e. for negative t) is small compared to K. Then there can be no consistent motion, because p, in the future, will be approximately equal to the impulse received at $t = 0$, and therefore p and this impulse cannot have opposite signs.

Indeed, consider the equation of motion

$$\dot{p}(t) = -K\delta(t)\,\epsilon[p(\tau)] \tag{2.1}$$

where τ is a positive number and ϵ is the sign function. (It would not matter if $\delta(t)$ were smeared over a small time-interval.) Integration of (2.1) gives

$$p_- - p_+ = K\epsilon(p_+) \tag{2.2}$$

where $p_\pm$ is the value of p for positive/negative t.

If we plot both sides of (2.2) as functions of p_+ (see Fig. 1) there is no consistent solution, if $|p_-| < K$.

However, a discontinuous force such as the one in equation (2.1) cannot exist in nature. If we replace the sign function by a continuous function of about the same shape, such as $\text{th}(Np_+)$ with large N, then we obtain Fig. 2. There is always a solution, where $p_+ \approx 0$, i.e. the signal is very weak.†

The same result would be obtained if we considered a discontinuous force like the one in Fig. 1, but acting with a *time delay*‡ if the signal which activates it is very weak (e.g., a force due to the action of an on/off switch).

† It is amusing to note that if we reverse the sign of K, we obtain, for small p, *three* different solutions: there are three distinct possible motions, all consistent with the same initial conditions.

‡ It is then essential that the delta function in (2.1) be smeared over a small time-interval, in addition to being delayed (for small $p(\tau)$).

As long as the system is closed and behaves according to well-defined equations of motion, the strength of the signal (i.e. the final value of p_+) and the time delay will adjust themselves so as to obtain a self-consistent solution.

This discussion effectively suggests an existence theorem for a certain kind of differential equation with advanced argument. The general message of this part of the paper is this: In any case where a signal from the future seems liable to tampering, either that signal will come through sufficiently muddled and unclear, or the overall system will otherwise conspire to subvert any schemes for tampering. More abstractly stated: reasonable initial conditions do not lead to inconsistencies. In the context of any particular model this amounts to the statement of an existence theorem for

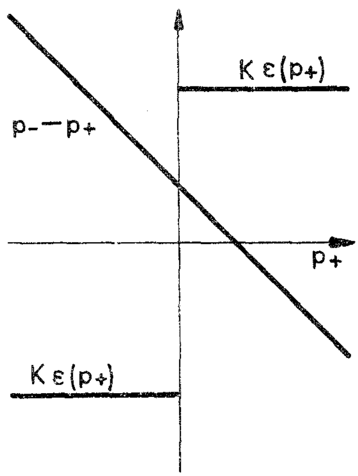

Figure 1.—Attempt at graphical solution of equation (2.2).

some set of differential equations with advanced arguments. Theorems of this kind are known (Bellman & Cooke, 1963; Elsgolts, 1966), often with conditions for uniqueness also, although as the footnote on page 378 indicates, one does not always have uniqueness. One should not be too demanding with regard to correct mathematical proofs, however, since even existence theorems for time symmetric electrodynamics, the system studied by Wheeler and Feynman, seem far beyond present mathematical techniques.† A rigorous existence proof along the lines suggested by the present paper has, however, recently been given (Peres and Schulman, 1972).

3. *A Counter-Example to the Above Solution*

Let us return for a moment to the original story (a mechanism getting a signal at 1 p.m., instructing it to destroy at 2 p.m. the emitter which sends

† Rigorous results in electrodynamics appear to exist only for retarded interaction without radiation reaction, in one-dimensional space. See Driver, 1962, 1965.

the signal at 3 p.m.). As explained, in the solution proposed by Wheeler and Feynman the signal is very weak, just at the threshold needed to operate the destroying mechanism. The latter does not act at 2 p.m., but rather just before 3 p.m., so that the emitter can still emit a weak signal before being destroyed.

We complicate the story by adding a second mechanism contrived in such a way as to force some *random* decision of the first one at 1.30 p.m., if the first mechanism has still not acted because the signal was ambiguous. For instance, the second mechanism may be a human instructed to operate the switch one way or the other at 1.30 p.m., if, by that time, it is uncertain whether or not a signal has been received (he may decide by 'free will', or

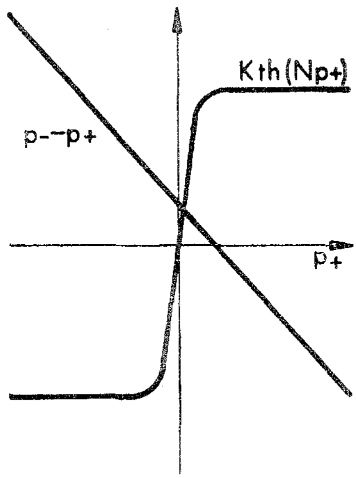

Figure 2.—Graphical solution of continuous version of equation (2.2).

by tossing a coin). We now appear to have a contradiction: e.g., if the coin decides not to destroy the emitter, then a strong signal will be emitted at 3 p.m. and therefore was received at 1 p.m., clearly directing the first mechanism to destroy the emitter.

To be sure, this contradiction can be resolved like that of equation (2.1), by invoking continuity. There is always a consistent motion, but then the idea of *randomness* is seriously compromised, as we presently show.

A model for the above situation is the equation

$$\dot{p}(t) = -K\{\delta(t)\,\mathrm{th}[Np(\tau)] + \tfrac{1}{10}\delta\left(t - \frac{\tau}{2}\right)\mathrm{Im}\,[\cosh Np(\tau) - 2]^{1/2}\} \quad (3.1)$$

with a *random* decision to pick one of the two branches of the square root. A short calculation shows the graph analogous to Figs. 1 and 2 for this

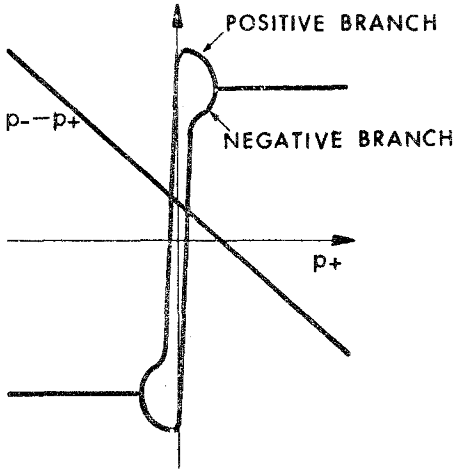

Figure 3.—Flipping a coin to force a decision.

equation to be Fig. 3. There are two possible values of p_+, depending on the branch chosen. As usual, both are very close to threshold.

But now, we run into an altogether different kind of problem. An observer can measure p_-, and at $t = \tau/10$ again measure p. For an appropriate range of initial conditions he can, using Fig. 3, deduce the sign of the square root in equation (3.1). That is, he can predict in advance how the coin will fall, even before the coin has been tossed (even if the coin has not yet been minted).

The above example shows that while advanced actions do not lead to inconsistencies in closed deterministic systems, they are incompatible with genuine random processes (or indeterminate processes such as human free will). In particular, if tachyons do exist, it is impossible to construct instruments emitting them in a controllable way.†

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† To be sure, the existence of 'true randomness' as described here is debatable and at the least ought to require an infinite number of degrees of freedom. This is presumably related to the question of whether our 'control' over events is illusory, if that term is used, as in the last sentence of this article, in the sense that Newton (1970) used it.

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